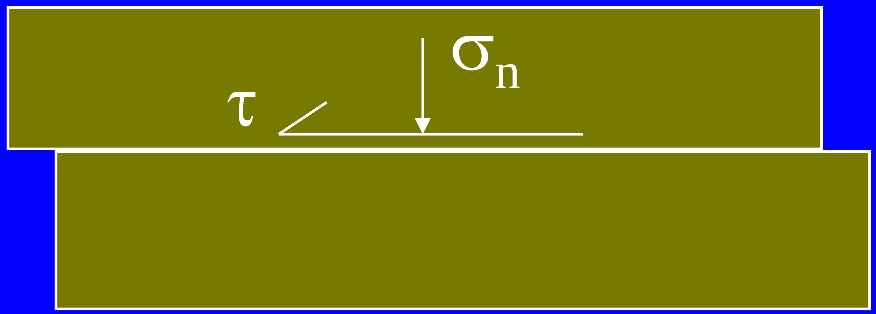
Soil Strength

Soil strength

- Soils are essentially frictional materials
  - the strength depends on the applied stress
- Strength is controlled by effective stresses
  - water pressures are required
- Soil strength depends on drainage
  - different strengths will be measured for a given soil that
    - (a) deforms at constant volume (undrained) and
    - (b) deforms without developing excess pore pressures (drained)

Mohr-Coulomb failure criterion



The limiting shear stress (soil strength) is given by

$$\tau = c + \sigma_n \tan \phi$$

where c = cohesion (apparent)

ϕ = friction angle

Mohr-Coulomb failure criterion

- The parameters c, ϕ are in general not soil constants. They depend on
- the initial state of the soil (OCR or I_d)
- the type of loading (drained or undrained)

- The Mohr-Coulomb criterion is an empirical criterion, and the failure locus is only locally linear. Extrapolation outside the range of normal stresses for which it has been determined is likely to be unreliable.

Effective stress failure criterion

If the soil is at failure the effective stress failure criterion will always be satisfied.

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c' and ϕ' are known as the effective (or drained) strength parameters.

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Soil behaviour is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters. But they are not necessarily soil constants.

Total stress failure criterion

If the soil is taken to failure at constant volume (undrained) then the failure criterion can be written in terms of total stress as

$$\tau = c_u + \sigma_n \tan\phi_u$$

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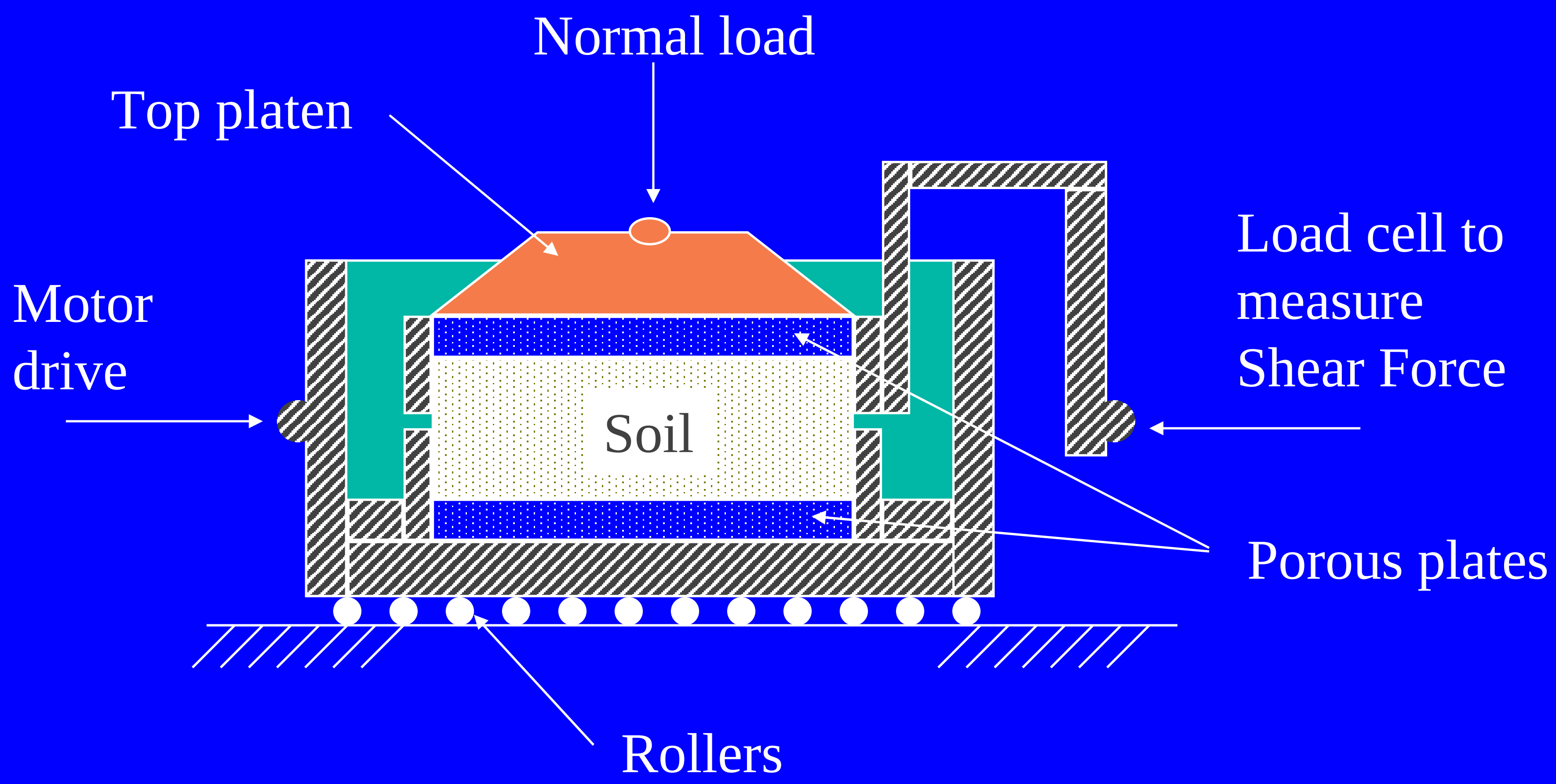
These parameters are not soil constants, they depend strongly on the moisture content of the soil.

The undrained strength is only relevant in practice to clayey soils that in the short term remain undrained. Note that as the pore pressures are unknown for undrained loading the effective stress failure criterion cannot be used.

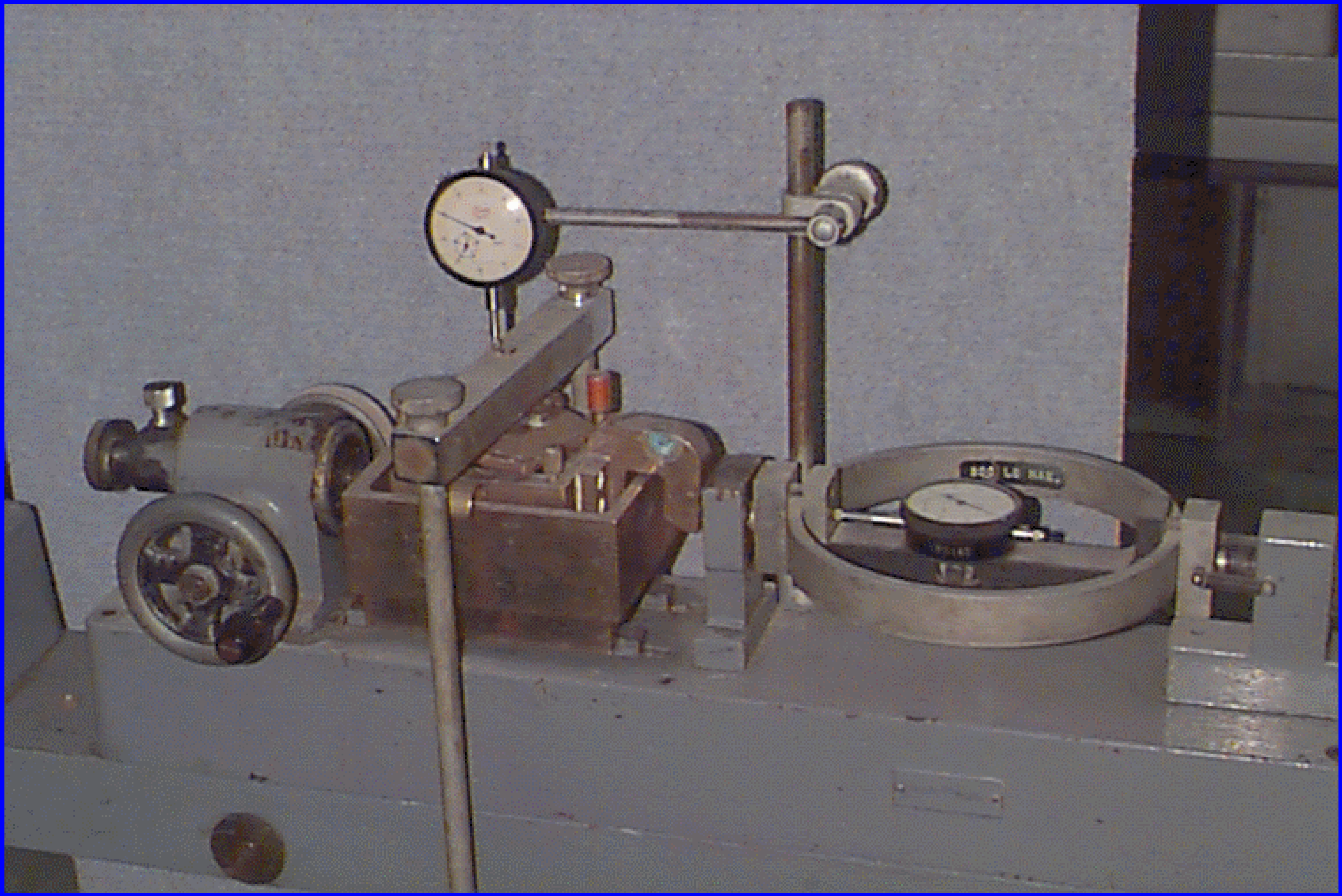
Tests to measure soil strength

1. Shear Box Test



Measure relative horizontal displacement, dx

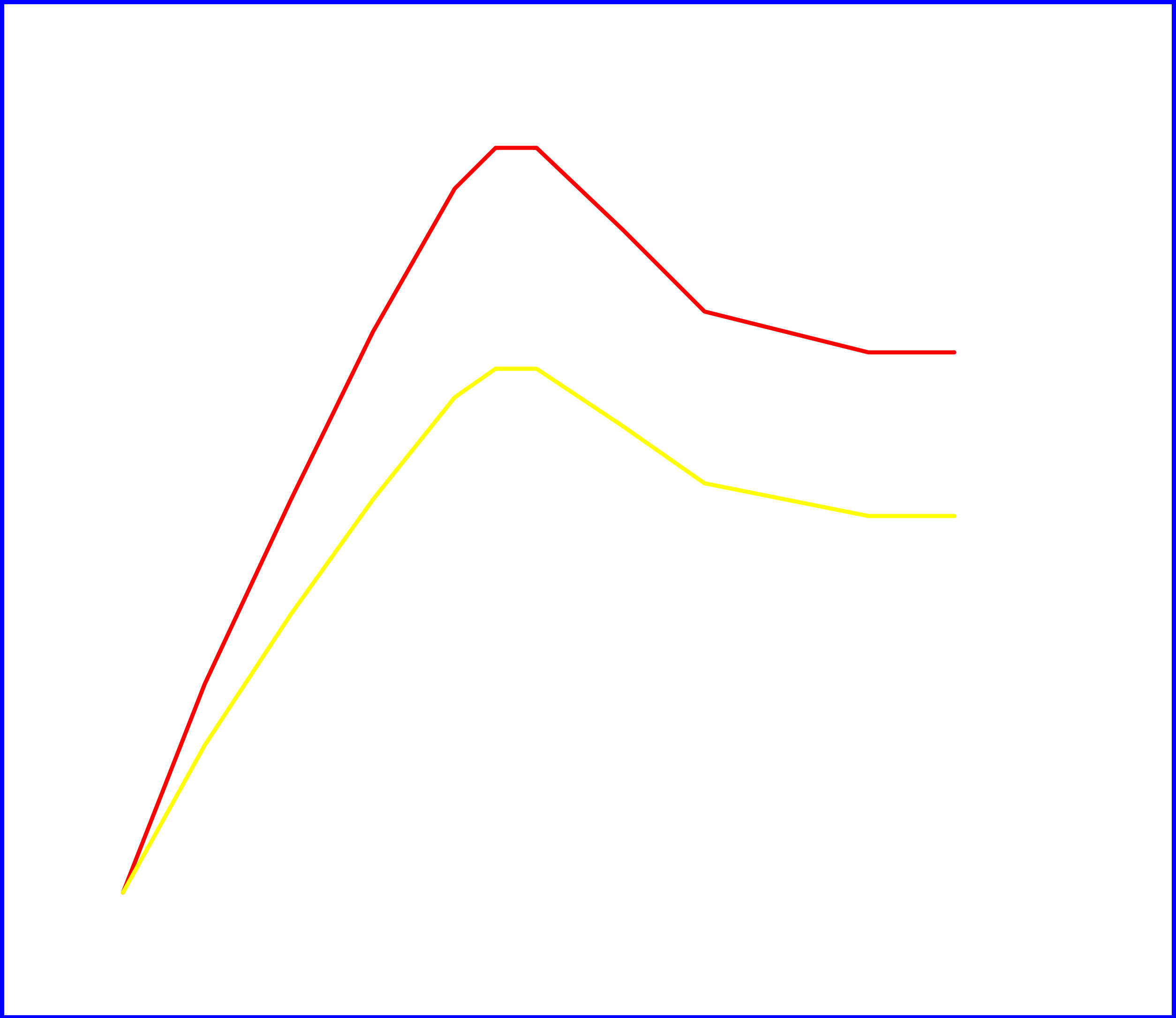
vertical displacement of top platen, dy



Shear box test

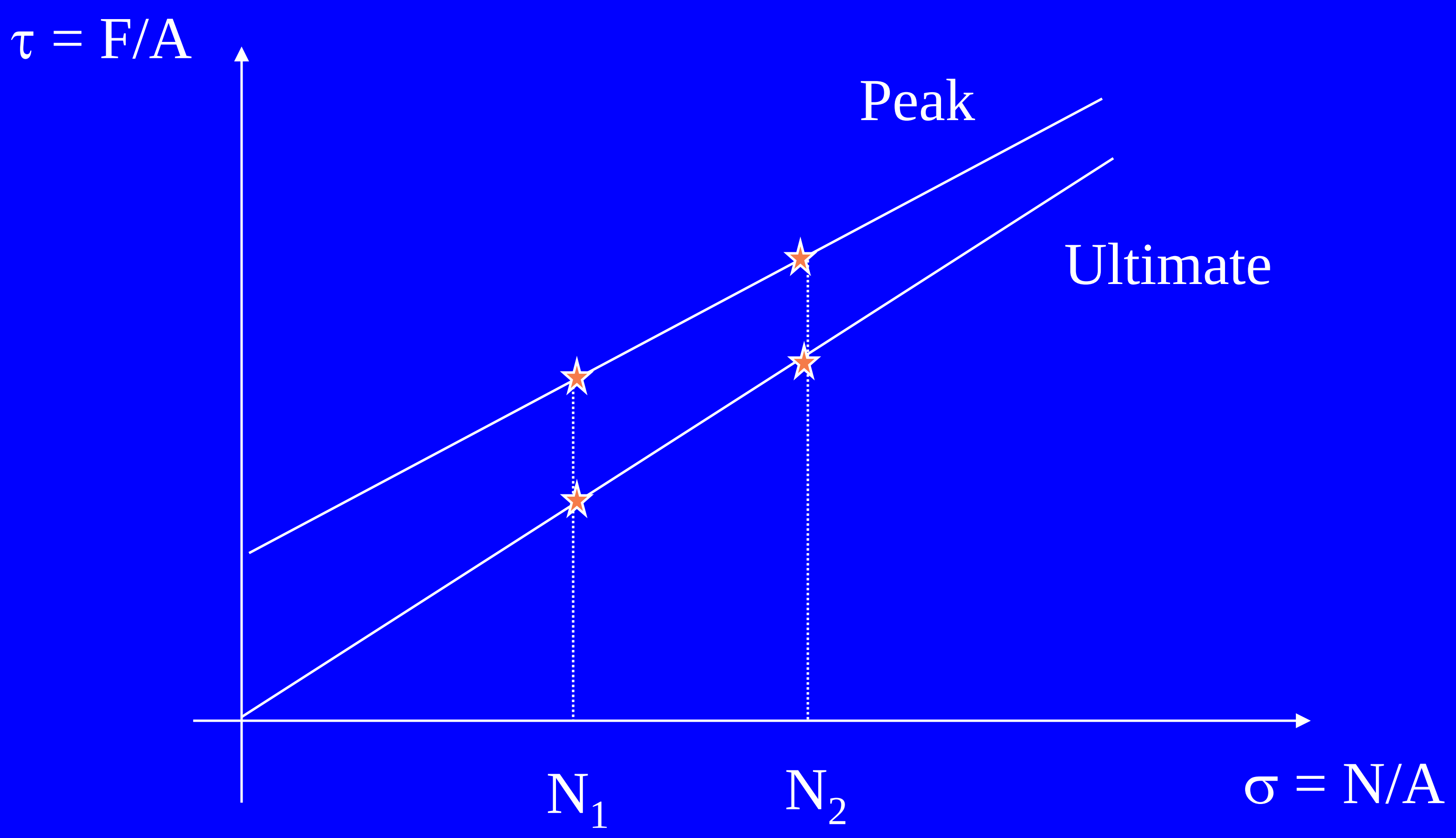
- Usually only relatively slow drained tests are performed in shear box apparatus. For clays rate of shearing must be chosen to prevent excess pore pressures building up. For sands and gravels tests can be performed quickly
- Tests on sands and gravels are usually performed dry. Water does not significantly affect the (drained) strength.
- If there are no excess pore pressures and as the pore pressure is approximately zero the total and effective stresses will be identical.
- The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters c', ϕ' can be determined.

Typical drained shear box results



Normal load increasing

Typical drained shear box results



Interpretation of shear box tests

- A peak and an ultimate failure locus can be obtained from the results each with different c' and ϕ' values.
- All soils are essentially frictional and continued shearing results in them approaching a purely frictional state where c' = 0.
- Normally consolidated clays (OCR=1) and loose sands do not show separate peak and ultimate failure loci, and for soils in these states c' = 0.
- Overconsolidated clays and dense sands have peak strengths with c' > 0.
- Note that dense sands do not possess any true cohesion (bonds), the apparent cohesion results from the tendency of soil to expand when sheared.

Shear box test - advantages

- Easy and quick test for sands and gravels
- Large deformations can be achieved by reversing shear direction. This is useful for determining the residual strength of a soil
- Large samples may be tested in large shear boxes. Small samples may give misleading results due to imperfections (fractures and fissures) or the lack of them.
- Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or other selected planes are required.

Shear box test - disadvantages

- Non-uniform deformations and stresses in the specimen. The stress-strain behaviour cannot be determined. The estimated stresses may not be those acting on the shear plane.

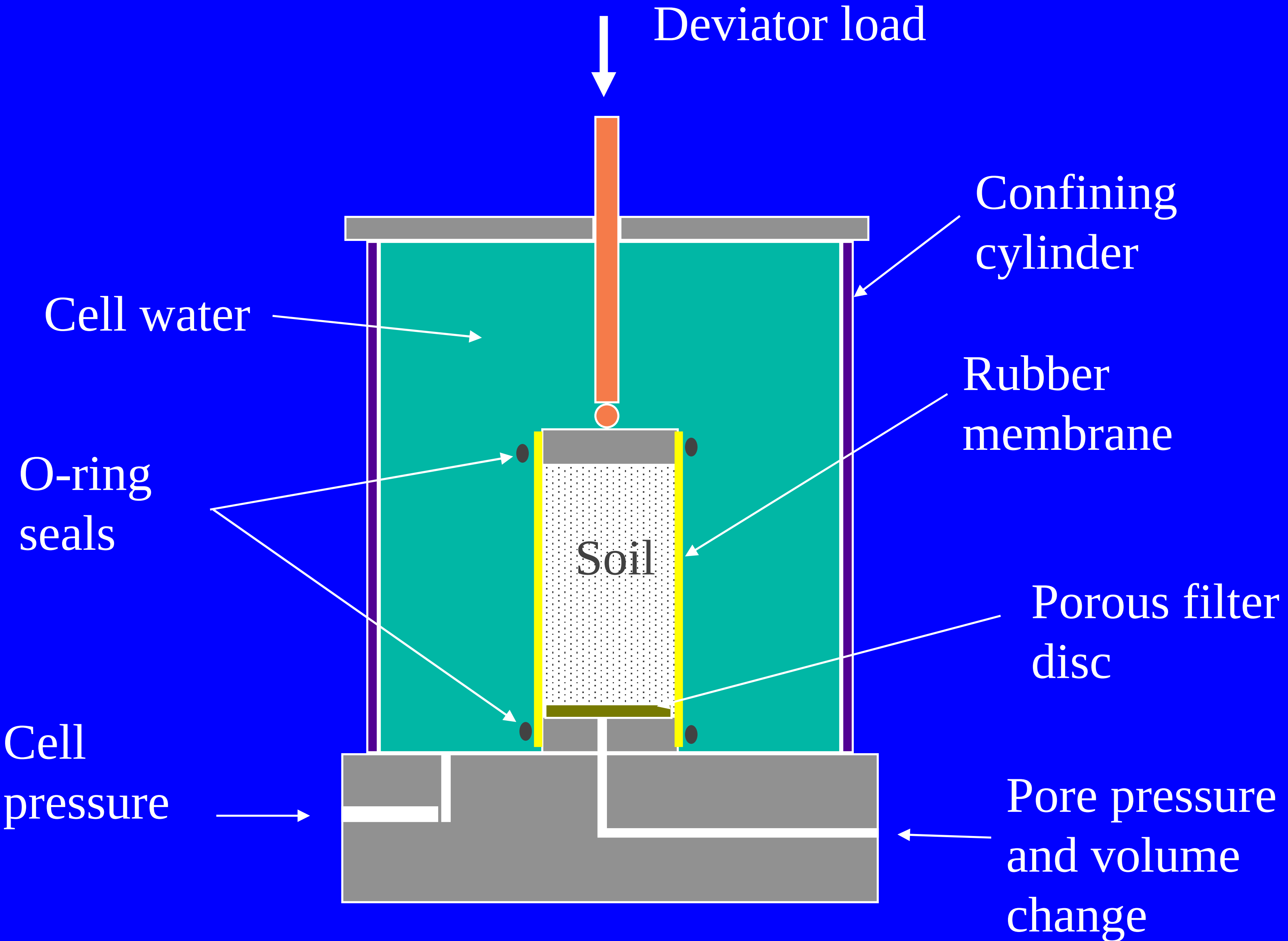
- There is no means of estimating pore pressures so effective stresses cannot be determined from undrained tests

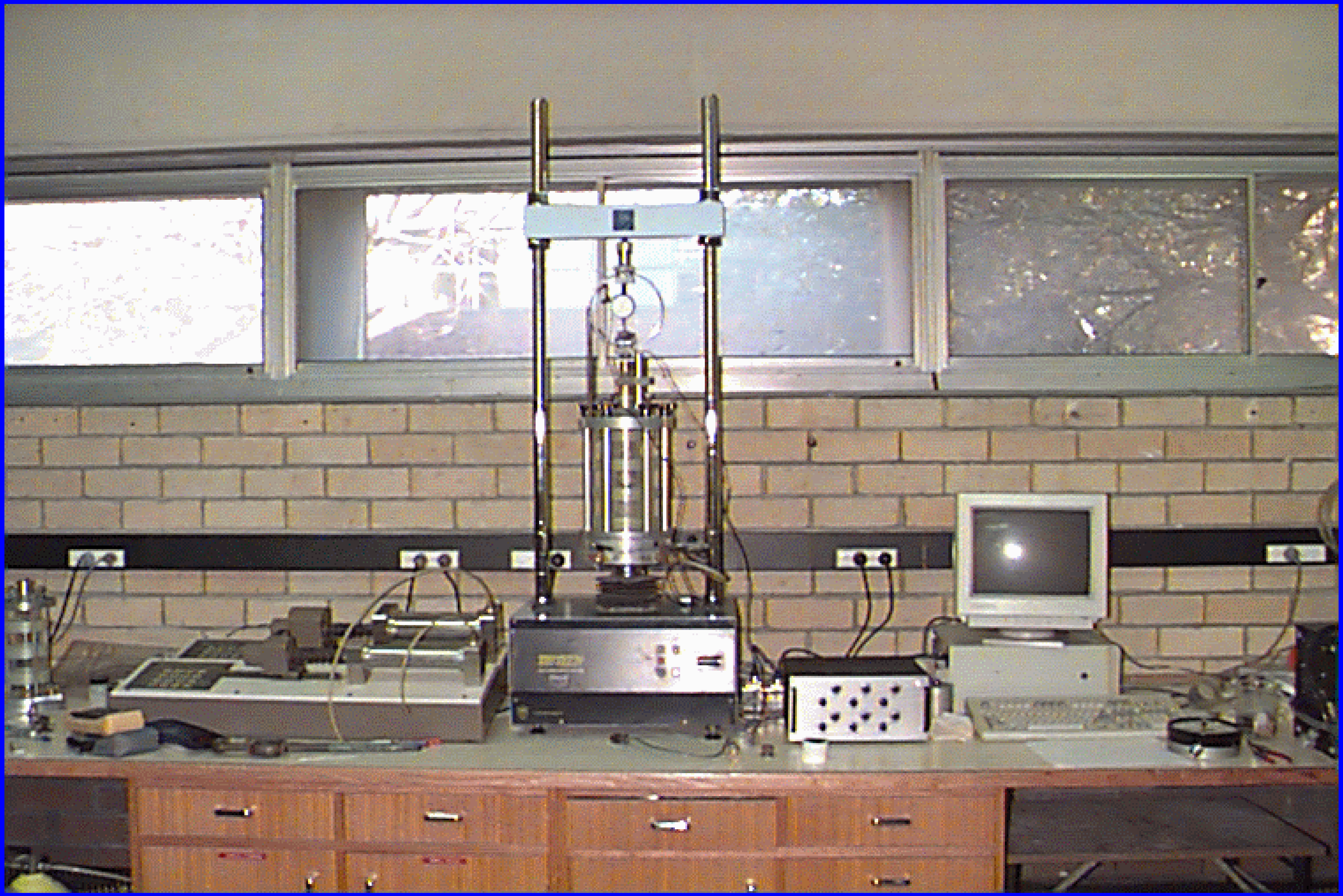
- Undrained strengths are unreliable because it is impossible to prevent localised drainage without high shearing rates

In practice shear box tests are used to get quick and crude estimates of failure parameters

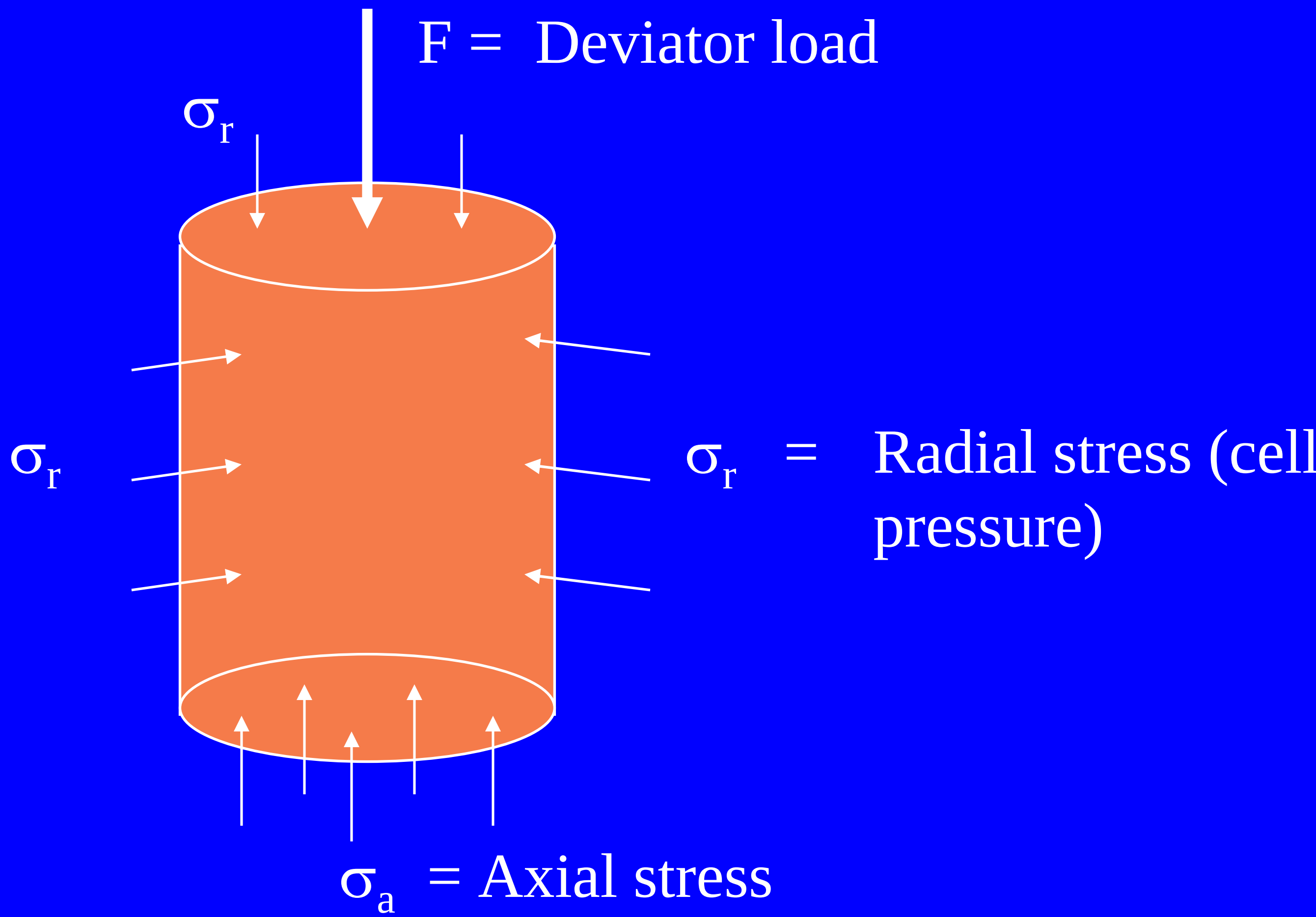
Tests to measure soil strength

2. The Triaxial Test



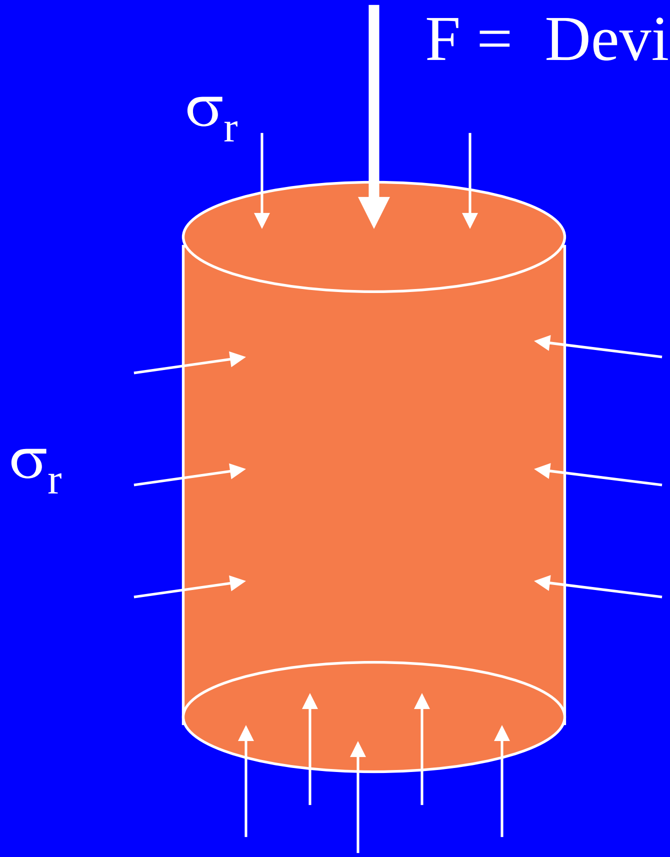


Stresses in triaxial specimens



Stresses in triaxial specimens

F = Deviator load



σ_r = Radial stress (cell pressure)

σ_a = Axial stress

From equilibrium we have $\sigma_a = \sigma_r + \dfrac{F}{A}$

Stresses in triaxial specimens

F/A is known as the deviator stress, and is given the symbol q

$$q = (\sigma_a - \sigma_r) = (\sigma_1 - \sigma_3)$$

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Increasing q is required to cause failure

Strains in triaxial specimens

From the measurements of change in height, dh, and change in volume dV we can determine

Axial strain $$\varepsilon_a = -\frac{dh}{h_0}$$

Volume strain $$\varepsilon_V = -\frac{dV}{V_0}$$

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where h_0 is the initial height and V_0 is the initial volume

It is assumed that the specimens deform as right circular cylinders. The cross-sectional area, A, can then be determined from

$$A = A_o \left(\frac{1 + \frac{dV}{V_0}}{1 + \frac{dh}{h_0}} \right) = A_o \left(\frac{1 - \varepsilon_v}{1 - \varepsilon_a} \right)$$

Types of triaxial test

There are many test variations. Those used most in practice are:

- UU (unconsolidated undrained) test.

  Cell pressure applied without allowing drainage. Then keeping cell pressure constant increase deviator load to failure without drainage.

- CIU (isotropically consolidated undrained) test.

  Drainage allowed during cell pressure application. Then without allowing further drainage increase q keeping σ_r constant as for UU test.

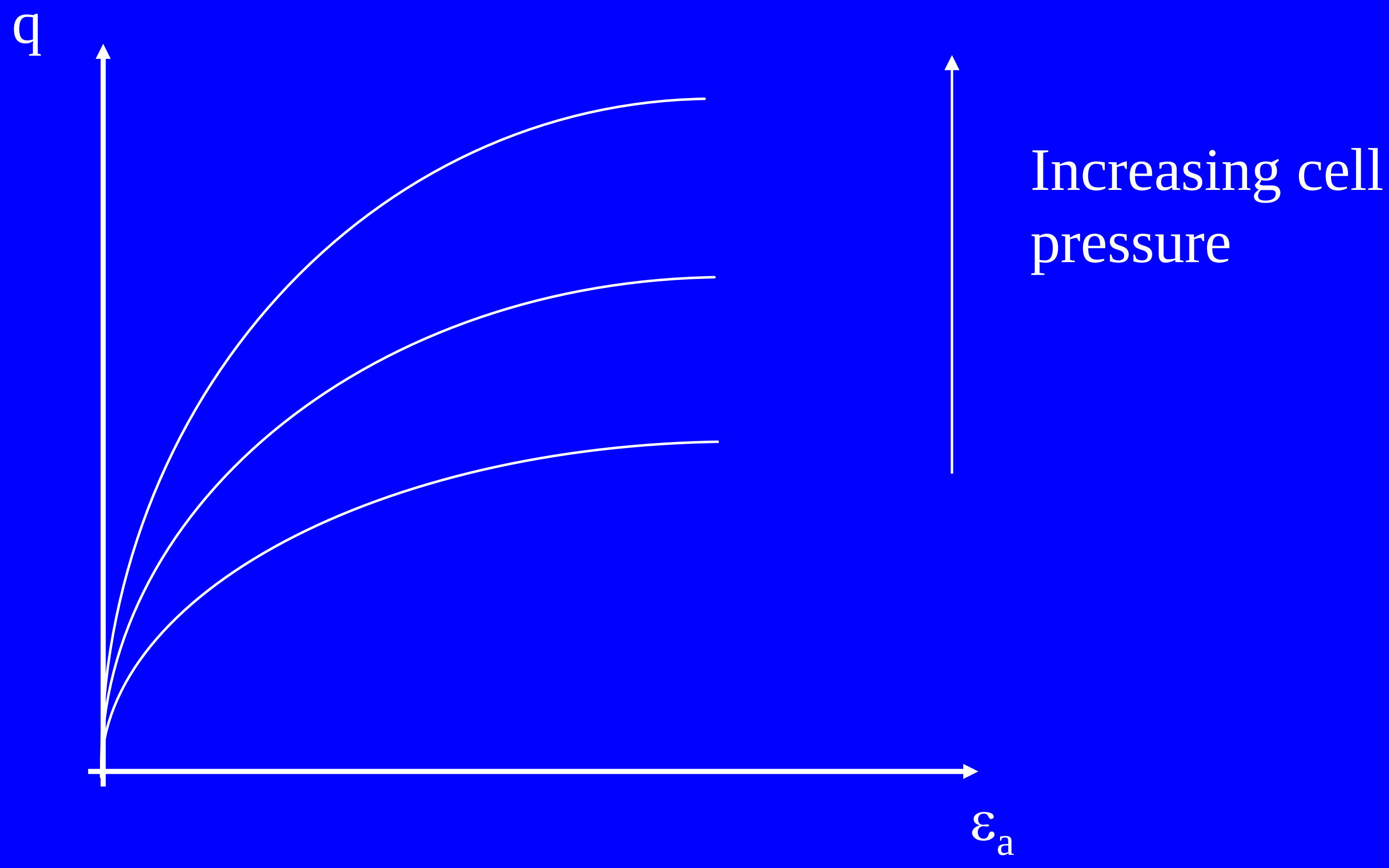
- CID (isotropically consolidated drained) test

  Similar to CIU except that as deviator stress is increased drainage is permitted.

Advantages of the triaxial test

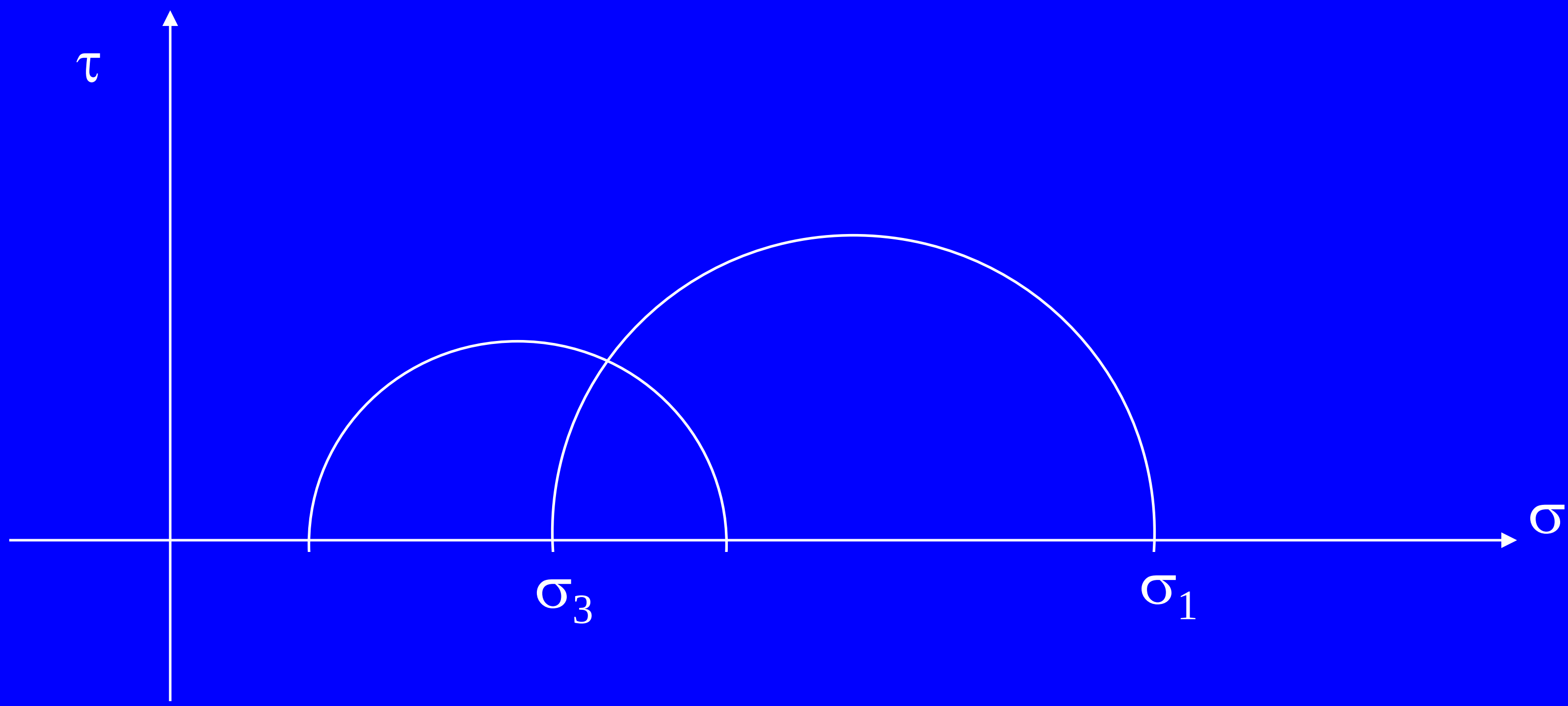
- Specimens are subjected to (approximately) uniform stresses and strains
- The complete stress-strain-strength behaviour can be investigated
- Drained and undrained tests can be performed
- Pore water pressures can be measured in undrained tests, allowing effective stresses to be determined
- Different combinations of cell pressure and axial stress can be applied

Typical triaxial results



Mohr Circles

To relate strengths from different tests we need to use some results from the Mohr circle transformation of stress.



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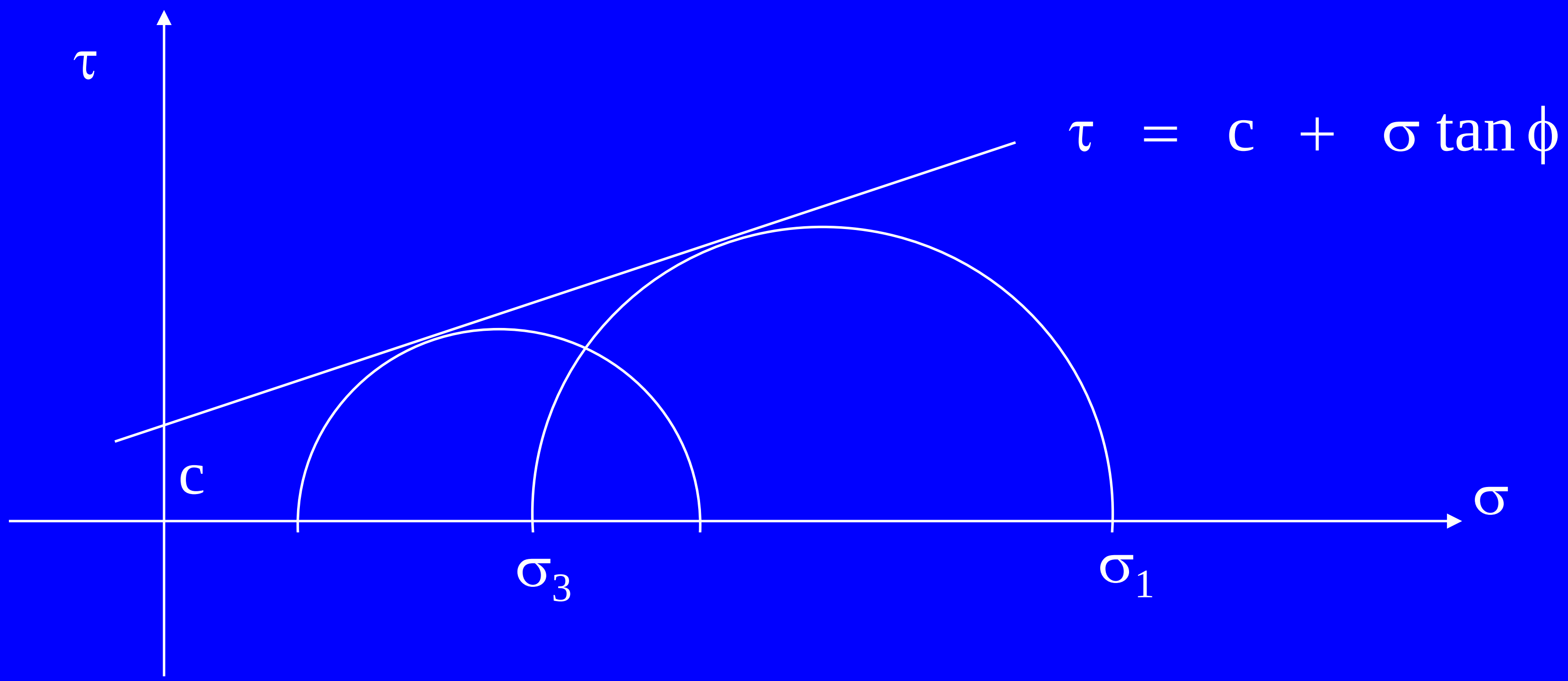
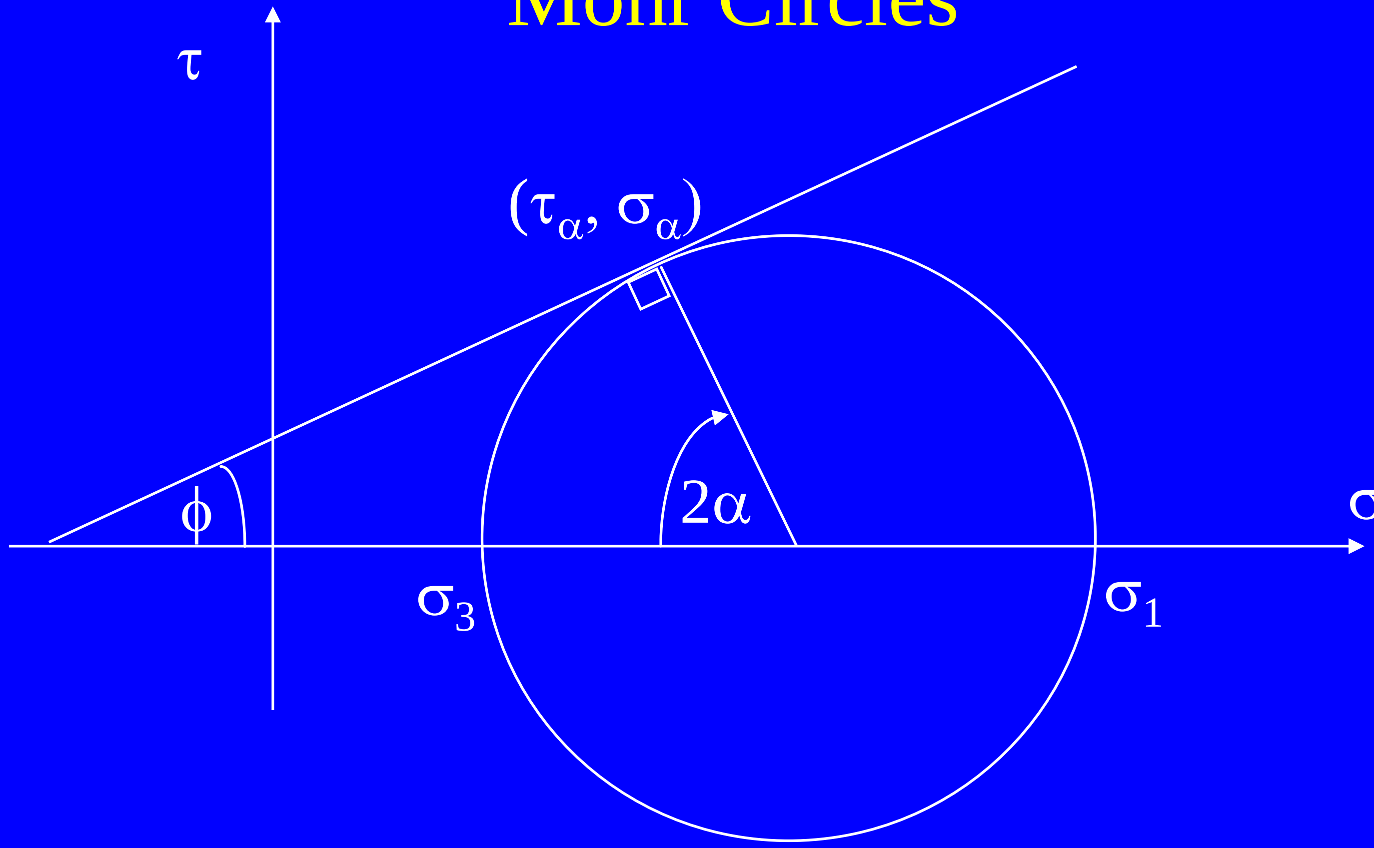


The Mohr-Coulomb failure locus is tangent to the Mohr circles at failure

Mohr Circles



From the Mohr Circle geometry

$$\sigma_\alpha = \frac{(\sigma_1 + \sigma_3)}{2} - \frac{(\sigma_1 - \sigma_3)}{2}\cos 2\alpha$$

$$\tau_\alpha = \frac{(\sigma_1 - \sigma_3)}{2}\sin 2\alpha$$

$$\alpha = \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

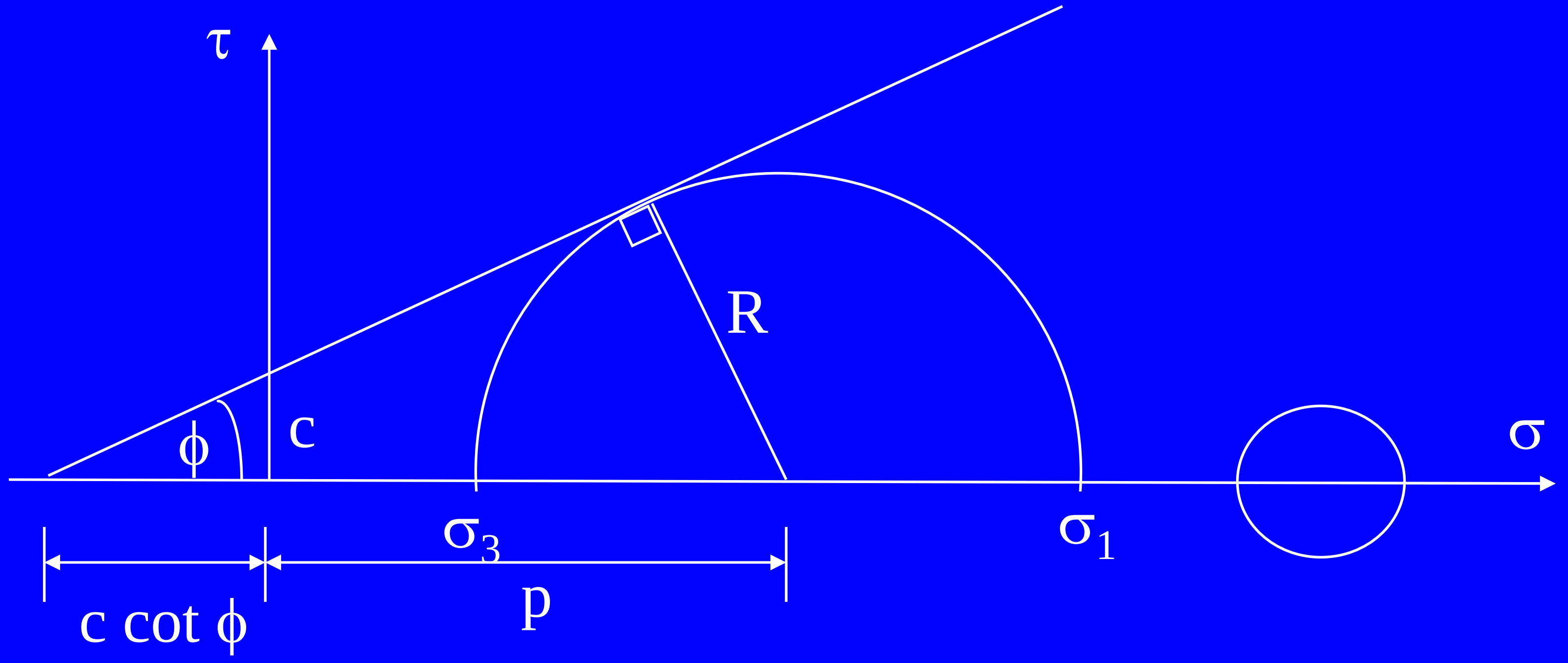
Mohr Circles

- The Mohr circle construction enables the stresses acting in different directions at a point on a plane to be determined, provided that the stress acting normal to the plane is a principal stress.
- The construction is useful in Soil Mechanics because many practical situations may be approximated as plane strain.
- The sign convention is different to that used in Structural analysis because it is conventional to take compressive stresses positive
- Sign convention: Compressive normal stresses positive

  Anti-clockwise shear stresses positive (from inside element)

  Angles measured clockwise are positive

Mohr-Coulomb criterion (Principal stresses)



Failure occurs if a Mohr circle touches the failure criterion. Then

$$R = \sin\phi \,(p + c \cot\phi)$$

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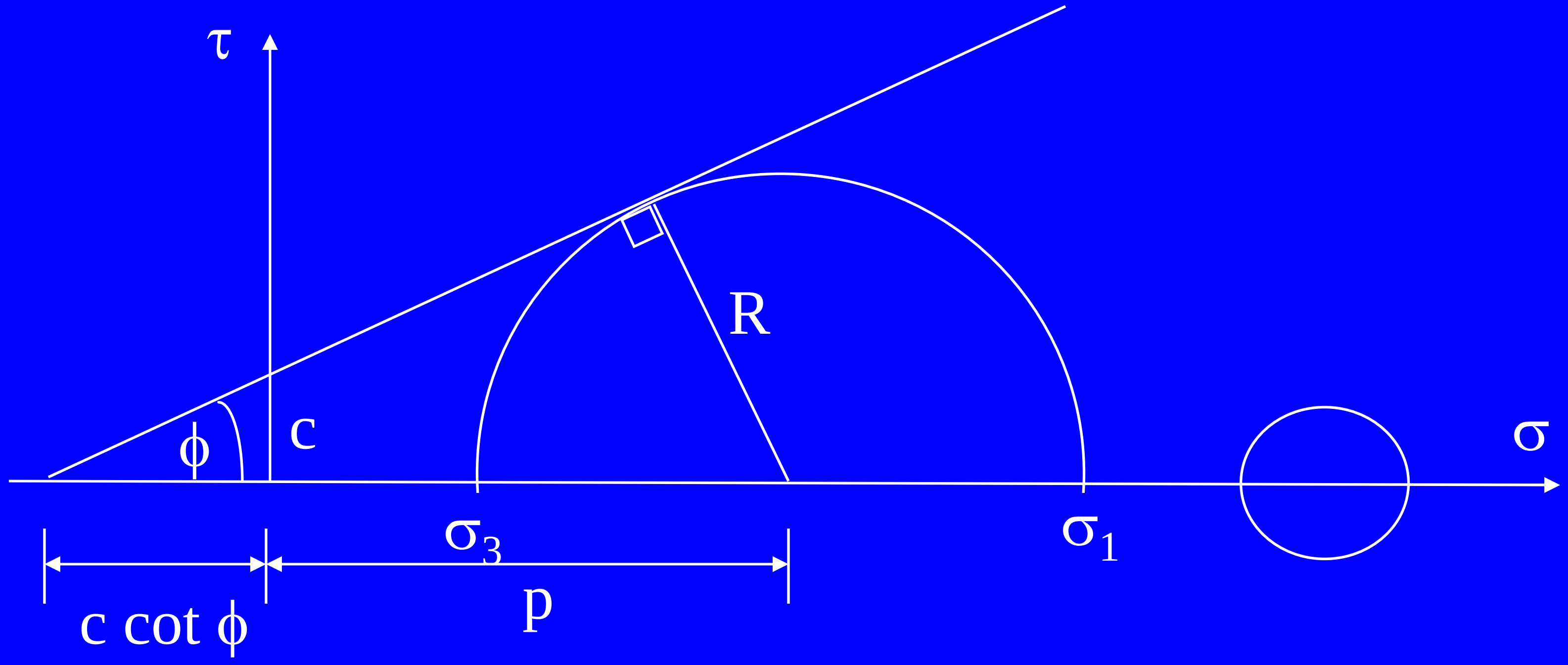

Failure occurs if a Mohr circle touches the failure criterion. Then

$$R = \sin\phi \, (p + c \cot\phi)$$

$$\frac{\sigma_1 + c \cot \phi}{\sigma_3 + c \cot \phi} = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left[\frac{\pi}{4} + \frac{\phi}{2} \right] = N_\phi$$

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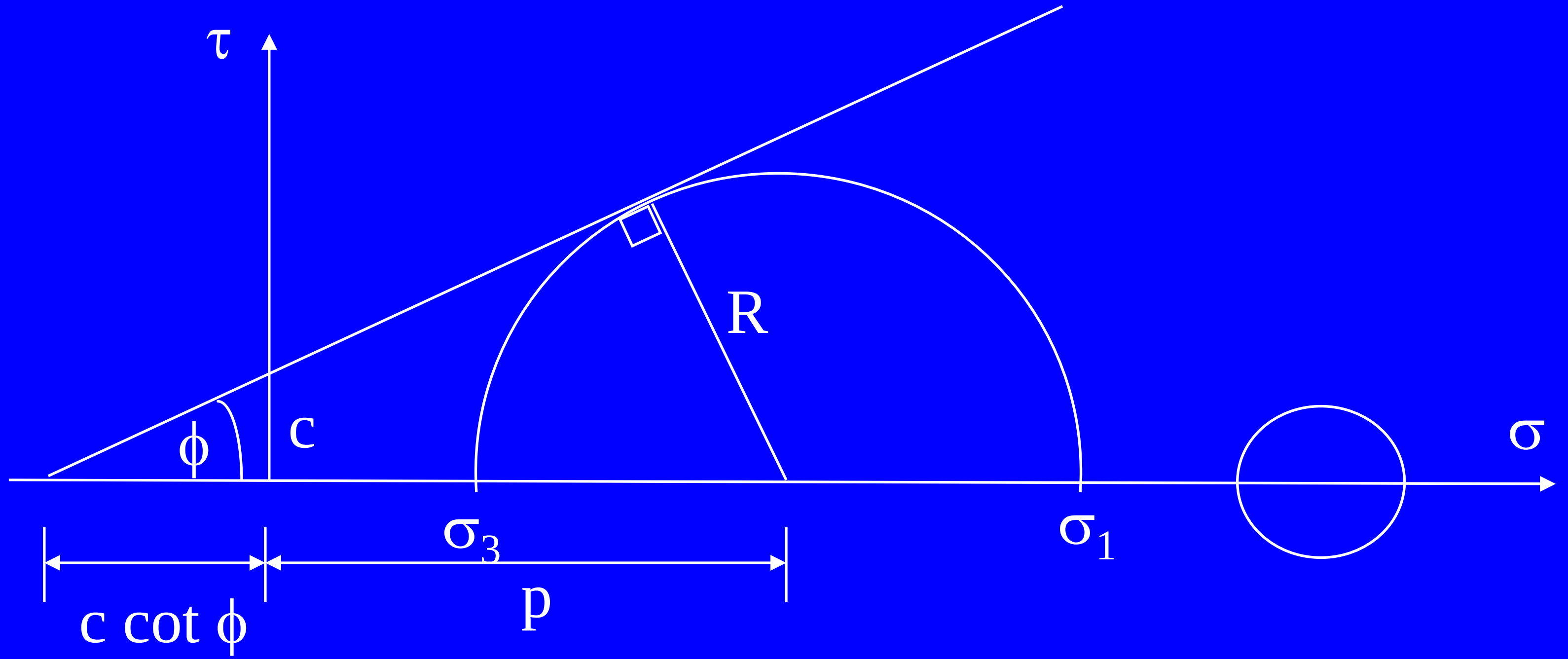

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$$\sigma_1 = N_\phi \, \sigma_3 + 2c\sqrt{N_\phi}$$

Effective stress Mohr-Coulomb criterion

As mentioned previously the effective strength parameters c' and ϕ' are the fundamental parameters. The Mohr-Coulomb criterion must be expressed in terms of effective stresses

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where

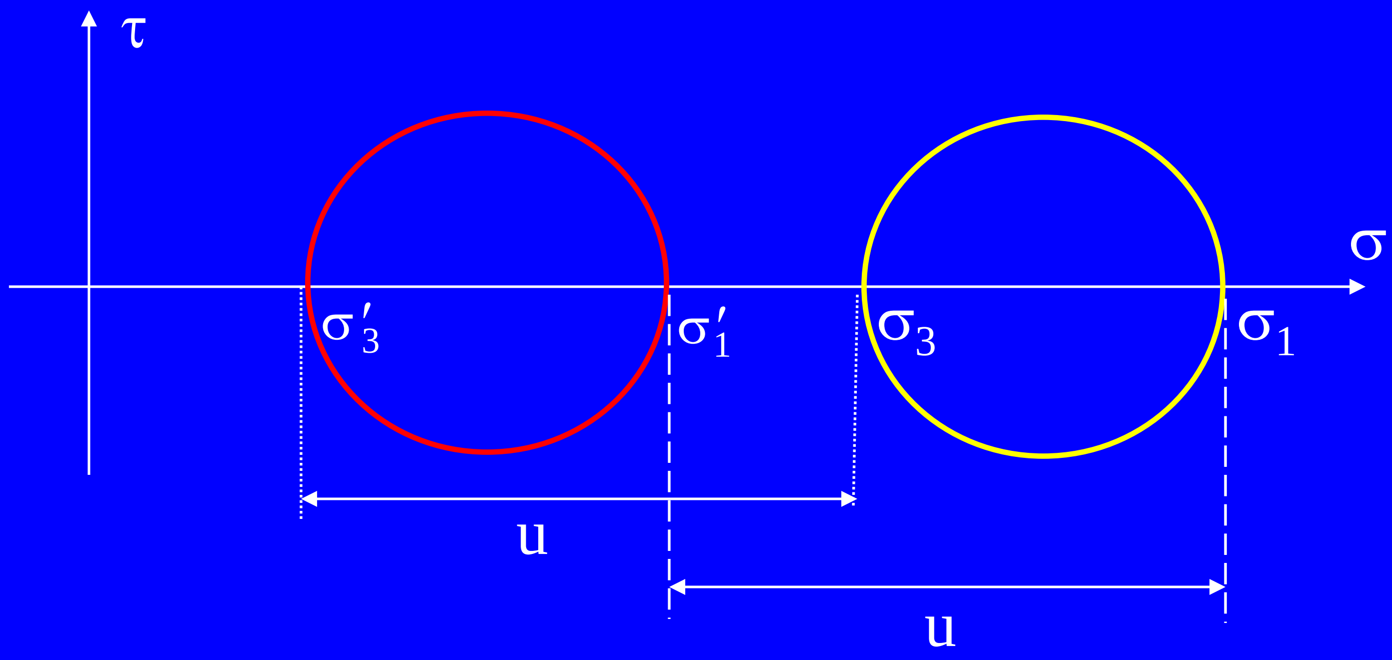
$$N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'}$$

$$\sigma'_n = \sigma_n - u$$

$$\sigma'_1 = \sigma_1 - u$$

$$\sigma'_3 = \sigma_3 - u$$

Effective and total stress Mohr circles



For any point in the soil a total and an effective stress Mohr circle can be drawn. These are the same size with

$$\sigma'_1 - \sigma'_3 = \sigma_1 - \sigma_3$$

The two circles are displaced horizontally by the pore pressure, u.

Interpretation of Laboratory results

1. Drained shear loading

- In laboratory tests the loading rate is chosen so that no excess water pressures will be generated, and the specimens are free to drain. Effective stresses can be determined from the applied total stresses and the known pore water pressure.
- Only the effective strength parameters c' and ϕ'have any relevance to drained tests.
- It is possible to construct a series of total stress Mohr circles but the inferred total stress (undrained) strength parameters are meaningless.

Interpretation of Laboratory results

- Effective strength parameters are generally used to check the long term stability (that is when all excess pore pressures have dissipated) of soil constructions.

- For sands and gravels pore pressures dissipate rapidly and the effective strength parameters can also be used to check the short term stability.

- In principle the effective strength parameters can be used to check the stability at any time for any soil type. However, to do this the pore pressures in the ground must be known and in general they are only known in the long term.

Interpretation of Laboratory results

2. Undrained loading

- In undrained laboratory tests no drainage from the sample must occur, nor should there be moisture redistribution within the sample.

- In the shear box this requires fast shear rates. In triaxial tests slower loading rates are possible because conditions are uniform and drainage from the sample is easily prevented.

- In a triaxial test with pore pressure measurement the effective stresses can be determined and the effective strength parameters c', ϕ' evaluated. These can be used as discussed previously to evaluate long term stability.

Interpretation of Laboratory results

- The undrained tests can also be used to determine the total (or undrained) strength parameters c_u, ϕ_u. If these parameters are to be relevant to the ground the moisture content must be the same. This can be achieved either by performing UU tests or by using CIU tests and consolidating to the in-situ stresses.
- The total (undrained) strength parameters are used to assess the short term stability of soil constructions. It is important that no drainage should occur if this approach is to be valid. For example, a total stress analysis would not be appropriate for sands and gravels.
- For clayey soils a total stress analysis is the only simple way to assess stability
- Note that undrained strengths can be determined for any soil, but they may not be relevant in practice

Relation between effective and total stress criteria

Three identical saturated soil samples are sheared to failure in UU triaxial tests. Each sample is subjected to a different cell pressure. No water can drain at any stage.

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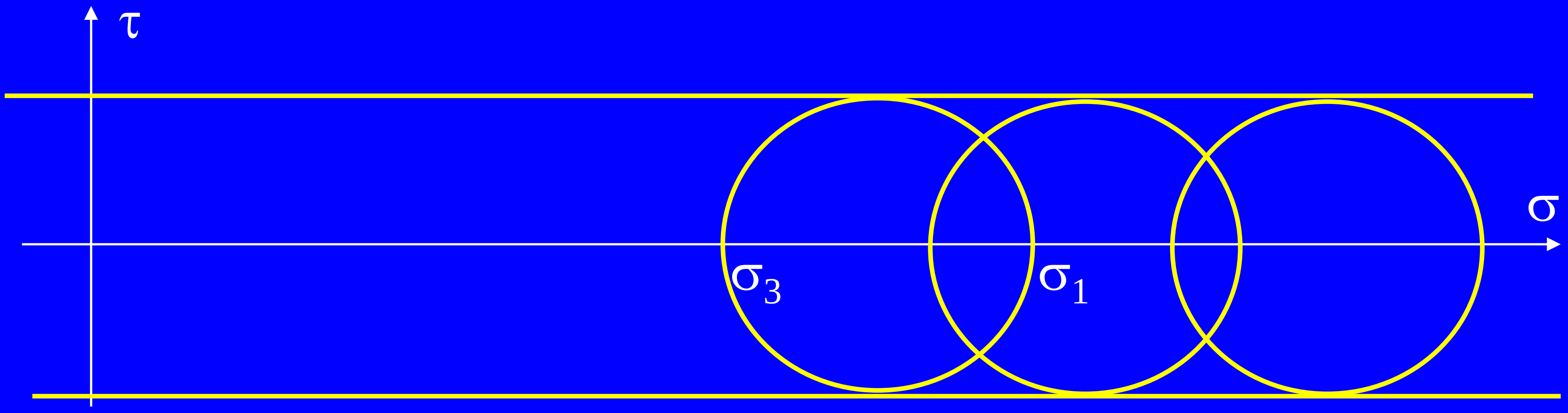

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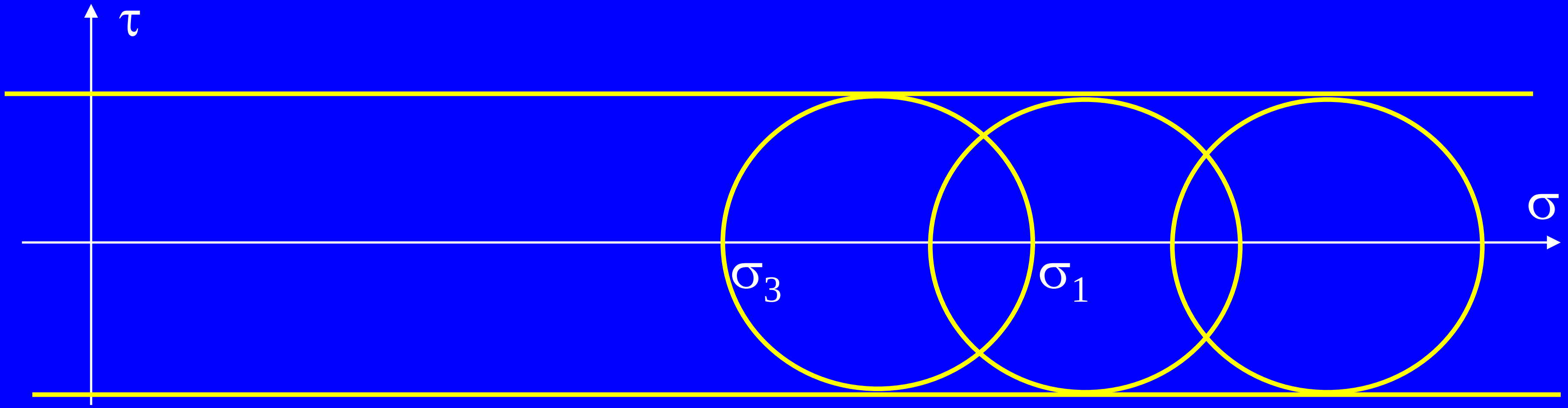
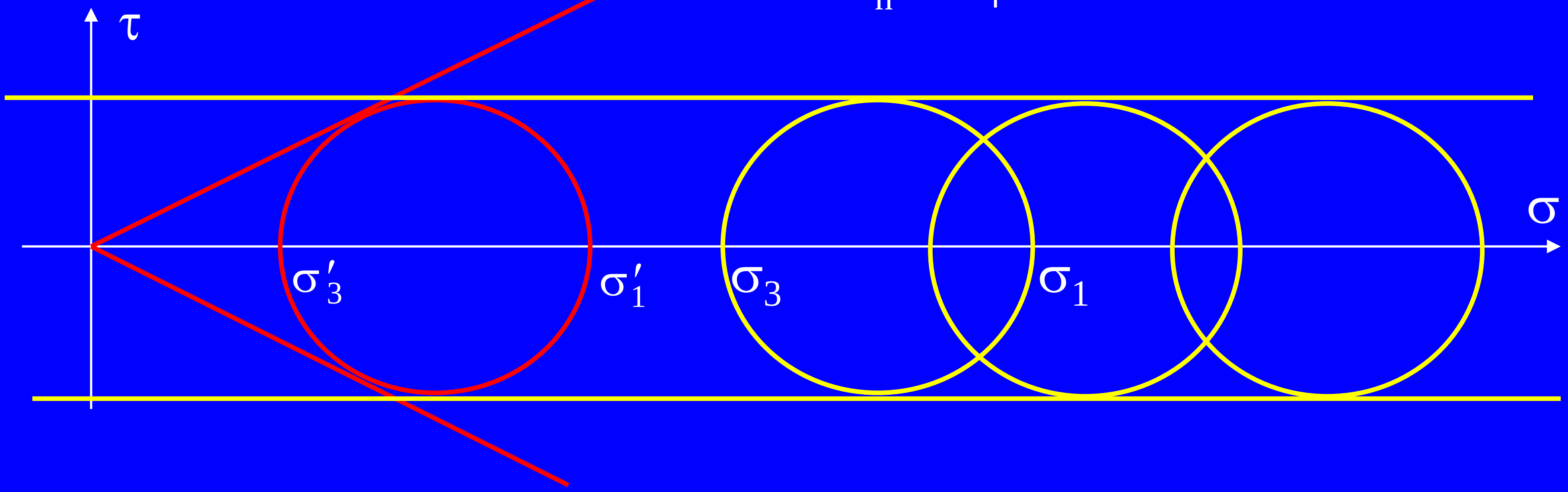


We find that all the total stress Mohr circles are the same size, and therefore $\phi_u = 0$ and $\tau = s_u = c_u = \text{constant}$

Relation between effective and total stress criteria

Because each sample is at failure, the fundamental effective stress failure condition must also be satisfied. As all the circles have the same size there must be only one effective stress Mohr circle

$$\tau = c' + \sigma_n' \tan\phi'$$



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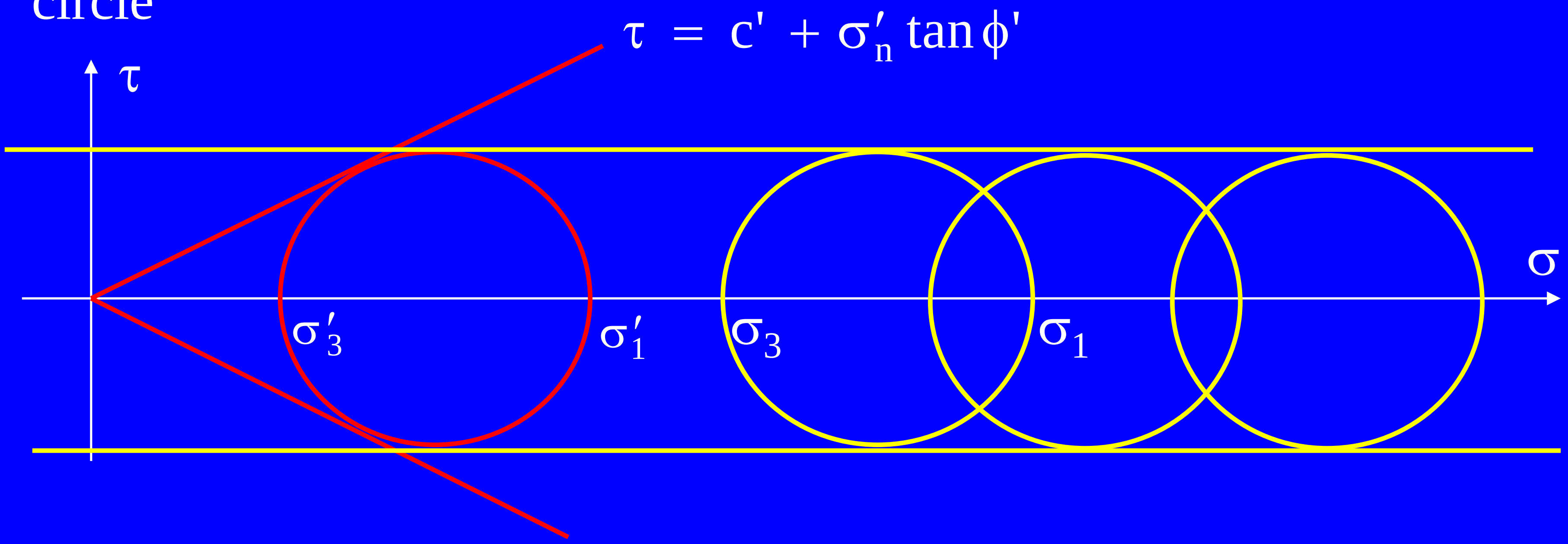

We have the following relations

$$\sigma_1' - \sigma_3' = \sigma_1 - \sigma_3 = 2 c_u$$

$$\sigma_1' = N_\phi \sigma_3' + 2 c' \sqrt{N_\phi}$$

Relation between effective and total stress criteria

- The different total stress Mohr circles with a single effective stress Mohr circle indicate that the pore pressure is different for each sample.

- As discussed previously increasing the cell pressure without allowing drainage has the effect of increasing the pore pressure by the same amount ($\Delta u = \Delta\sigma_r$) with no change in effective stress.

- The change in pore pressure during shearing is a function of the initial effective stress and the moisture content. As these are identical for the three samples an identical strength is obtained.

Significance of undrained strength parameters

- It is often found that a series of undrained tests from a particular site give a value of ϕ_u that is not zero (c_u not constant). If this happens either
  - the samples are not saturated, or
  - the samples have different moisture contents

- If the samples are not saturated analyses based on undrained behaviour will not be correct

- The undrained strength c_u is not a fundamental soil property. If the moisture content changes so will the undrained strength.

Example

In an unconsolidated undrained triaxial test the undrained strength is measured as 17.5 kPa. Determine the cell pressure used in the test if the effective strength parameters are c’ = 0, ϕ' = 26^o and the pore pressure at failure is 43 kPa.

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Analytical solution

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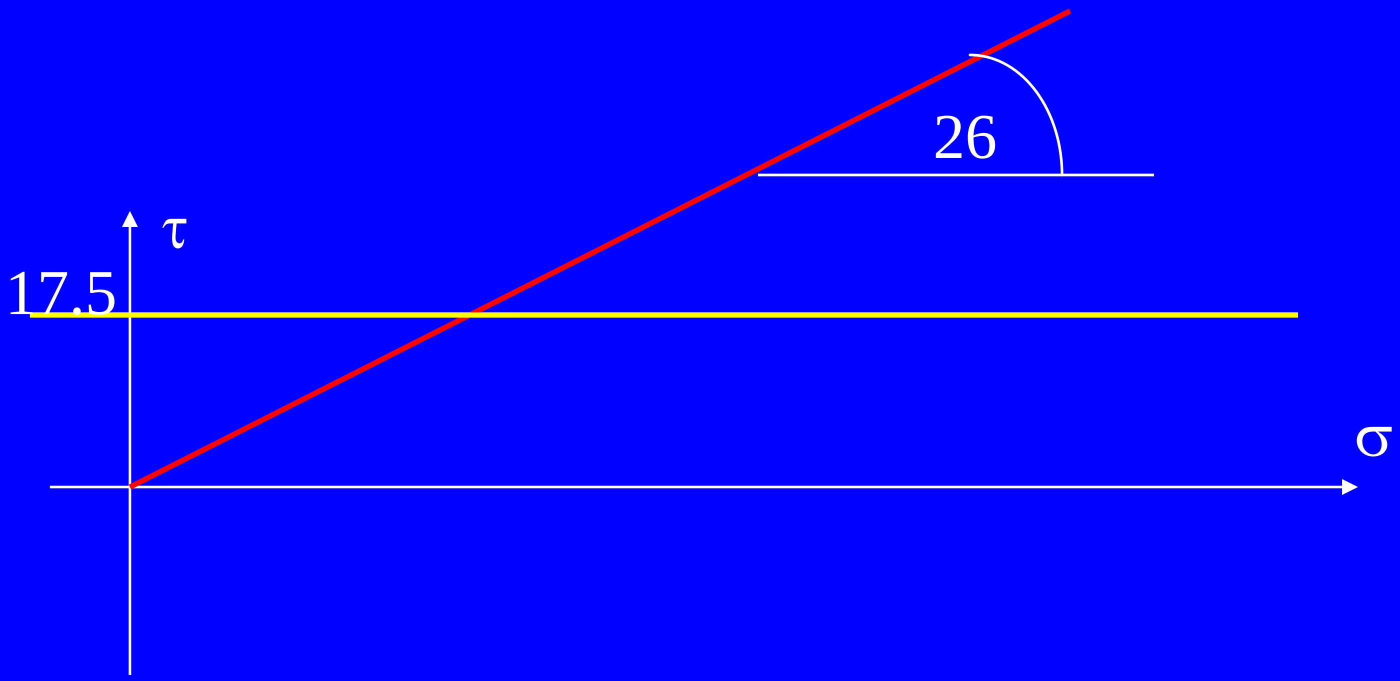
Hence $\sigma_1' =$ 57.4 kPa, $\sigma_3' =$ 22.4 kPa

and cell pressure (total stress) = $\sigma_3' + u =$ 65.4 kPa

Graphical solution



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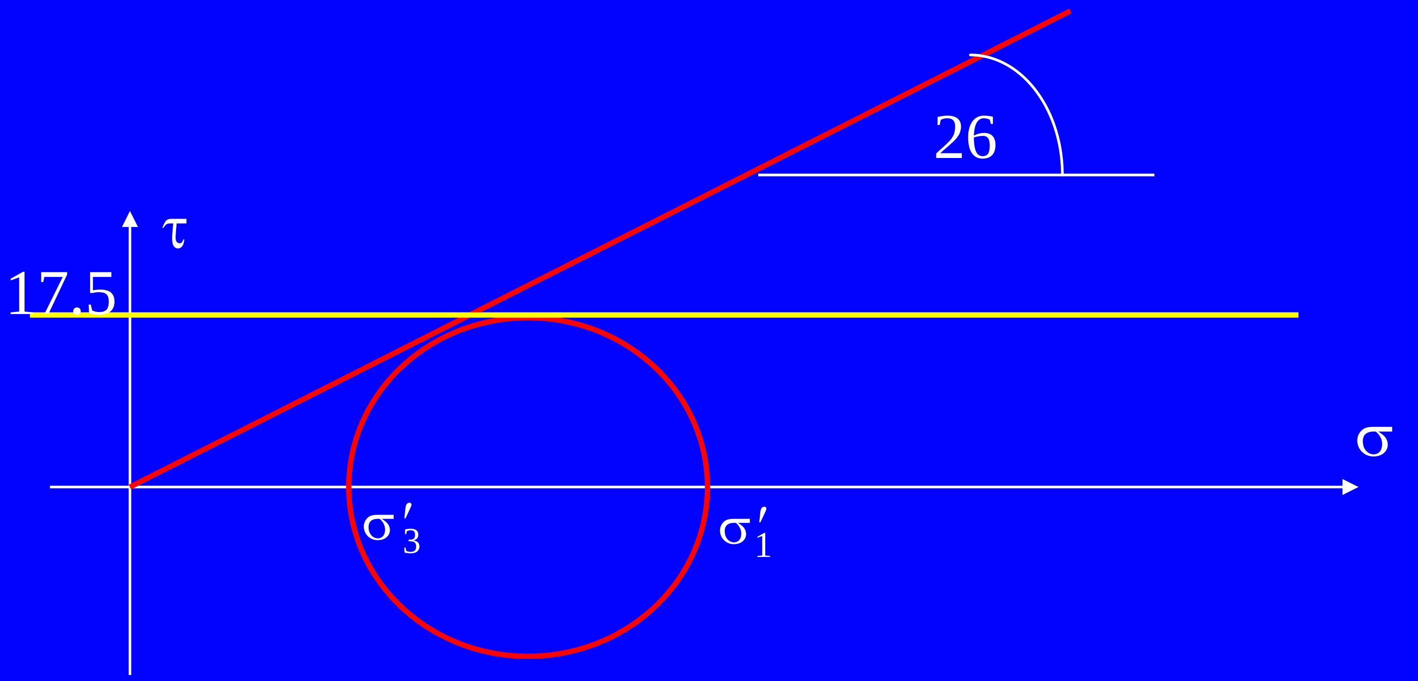

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